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Innovative Applications of OR

A dynamic principal-agent framework for modeling the performance of infrastructure

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ABSTRACT

This paper presents a novel approach to the problem of infrastructure development by integrating technical, economic and operational aspects, as well as the interactions between the entities who jointly carry out the project. The problem is defined within the context of a Public Private Partnership (PPP), where a public entity delegates the design, construction and maintenance of an infrastructure system to a private entity. Despite the benefits of this procurement method, the relationship between the two entities is inherently conflictive. Three main factors give rise to such conflict: the goals of the public and private party do not coincide, there is information asymmetry between them and their interaction unfolds in environments under uncertainty. The theory of contracts refers to this problem as a principal-agent problem; however, due to the complexity of the problem, it is necessary to recreate a dynamic interaction between the principal (i.e., the public entity) and the agent (i.e., the private entity) while including the monitoring of the infrastructure performance as an essential part of the interaction. The complex relationship between the sequential actions of players and the time-dependent behavior of a physical system is explored using a hybrid agent-based simulation model. The model is illustrated with several examples that show the versatility of the approach and its ability to accommodate the different decision strategies of the players (i.e., principal, agent) and the model of a physical infrastructure system.

1. Introduction

1.1. Infrastructure development

Since infrastructure systems are conceived to serve basic necessities of society, public institutions are responsible for their creation and persistence. However, the processes that entail its development are complex and sometimes public institutions are not prepared to manage them efficiently. This situation has paved the way for procurement methods where these complex tasks are delegated to specialized private third parties who are able to inject private capital investment and deal with complex technical aspects of design, construction and maintenance. Today, one of the most widely used category of this kind of delegation is the Public-Private Partnership (PPP) (Hoppe, Kusterer, & Schmitz, 2013; Kwak, Chih, & Ibbs, 2009; Levy, 2008; Yescombe, 2007).

The World Bank defines a PPP as a medium to long term arrangement where the public sector (e.g., a government agency) delegates some services or works to the private sector (e.g., a private firm), having agreed on objectives and conditions for the delivery. For clarity and consistency with other literature on the subject, we will refer to the government agency as the *principal* and treat it with female gender. The private firm will be referred to as the *agent*, with male gender. The services or works delegated to the agent are often either the enhancement of existing infrastructure or the design and construction of new infrastructure. Once this is completed, the public works are transferred temporarily to the agent—usually for a period ranging between 10 and 30 years—in which he assumes the responsibility of maintaining (i.e., performing maintenance works or updates to counteract deterioration) and operating the infrastructure (i.e., carrying out all the logistics necessary to provide the intended service) while receiving the rent produced by its operation. The agent also agrees to share risks with the principal. Those risks are related to design and construction costs, market demand, service and maintenance costs. It is common that in order to make the project economically attractive to the agent, the principal must provide subsidy: a payment schedule transferred to the agent during the infrastructure's operation. In order to ensure that the mentioned 'objectives and conditions' of the arrangement are fulfilled, the principal will keep track of certain performance indicators of the infrastructure by executing

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inspections. At the end of this contracted period, the government takes back control of the infrastructure system.

The central thesis that we want to convey in this paper is that the history and the success—or failure—of an infrastructure project featuring delegation (such as a PPP) results from the interplay of all the following aspects:

1. the economic game between the principal and the agent,
2. the regulatory framework and contractual design that constraint their interaction,
3. the performance of physical infrastructure, and
4. the natural environment in which the infrastructure is embedded.

In practice, a systematic framework that integrates these aspects to inform all the decisions involved in the delegation does not exist. We want to propose a model for such framework with the aim of showing the mechanisms by which these four aspects influence the result of the interaction. Additionally, we present suggestions that could transform the model into a decision support system for government agencies.

1.2. Public-private-partnership contracts

Even though the term *partnership* suggests that principal and agent are united by a legal partnership, and have intrinsic motivation for cooperating to achieve the greater good, this is not necessarily the case. As Yescombe points out (Yescombe, 2007, p.3), 'partnership' in this context is mostly a political slogan. In the use of a PPP procurement method the following circumstances are likely to appear:

- *Information asymmetry*: this is mainly caused because during the contractual relationship, the agent's actions are generally unobservable to the principal. Then, the agent knows his own level of effort and, therefore, can predict the infrastructure performance much better than the principal. On the other hand, the principal does not know how much effort the agent has employed in maintenance interventions and he can only estimate the performance of the infrastructure by actively inspecting it.
- *Conflicting goals*: this occurs because the objectives of both the agent and the principal lead to an adversarial relationship. Then, while the principal wants to reach a specific monetary balance and maximize some performance measure of the infrastructure, the agent simply wants to maximize his monetary balance.
- *Stochasticity*: the physical infrastructure system is fundamentally a stochastic system. Thus, from the point of view of a player (principal or agent), the merit of an action to be deployed at the present time instant is uncertain.

The act of delegation when these three features exist creates a *moral hazard*. In economics, the term moral hazard describes the situation in which an individual with private information is willing to take greater risks because someone else bears with the consequences. In our particular problem, the agent is willing to take risks by not doing a proper maintenance because the principal won't be aware of it and she is the one who will suffer from a low infrastructure performance.

The regulatory framework and the contractual design of the interaction is the main leverage point that the principal can use to control the moral hazard problem. In the literature dedicated to contracts in infrastructure projects we could identify two approaches. The first approach (Auriol & Picard, 2013; Medda, 2007) is deductive and quantitative. It is based on economics and game theory and uses closed form representations for the idealized interaction of fully rational economic agents. It deals with information constraints, risk preferences, utility functions and optimization problems. It is fundamentally prescriptive. It is able to produce precise quantitative specifications at the cost of reducing the complexity by imposing overly simplified assumptions on the problem so that it becomes mathematically tractable. This approach addresses the first component of our thesis (Section 1.1). The second approach (Meunier & Quinet, 2010; Yescombe, 2007) is inductive, and mostly qualitative. It is focused on the interface of finance, regulation and institutions. It is often sustained by experience, the extrapolation from past events and guided by subjective opinion. This approach is fundamentally descriptive. It makes reference to the minute details that involve the formation and persistence of a PPP. However, while being empirical and close to concrete examples, it often lacks the ability to produce a rigorous prescription of contract design. This approach addresses the second component of our thesis (Section 1.1).

Neither the first nor the second approach address the third and fourth components of our model (see Section 1.1) since they do not model the problem as a dynamic and path dependent interaction. Furthermore, they both overlook the fact that the physical system deteriorates over time, which would in turn elicit reactive actions from players. In summary, economic and management research on this topic has not studied the consequences of their principles in the context of a physical reality that influences players. Nevertheless, research on deterioration models for many kinds of civil infrastructure assets do exist (e.g., Frangopol, Kallen, & Noortwijk, 2004; Kleiner & Rajani, 2001; Kumar, Cline, & Gardoni, 2015; Sanchez-Silva, Klutke, & Rosowsky, 2011). These models effectively make the connection between: the properties of physical objects that compose the infrastructure system, the operations exerted on them, the pressures and demands coming from their environment and the resultant change in physical condition measured with some performance index.

1.3. Objective and scope

In this paper we propose a framework to designing contracts based on a reliable, reproducible quantitative model that acknowledges the intricate details of real economic and operational interactions and the inevitable deterioration of a physical infrastructure system under environmental pressures. For that purpose, we will develop an agent-based simulation model capable of tracing an interaction history between the principal, the agent, the natural environment and its effect on the infrastructure system. From such interaction we will calculate the utility for each player, which will rate the goodness of the delegation relationship that emerges out of certain player's strategies and problem parameters.

This paper is organized as follows. In Section 2, we present the traditional principal-agent framework and highlight its limitations. In Section 3 we propose an alternative interaction game that is the basis of the simulation model. In Section 4 a conceptual model of the interaction process is presented as a dynamic system, making explicit the dependence relationships. Then, the mathematical formulation of the model is described in detail in Section 5. The broad characteristics of the implementation in the form of a hybrid agent-based model are explained in Section 6. In Section 7 we present a set of numerical experiments and further analyses that highlight relevant aspects of the model. In Section 8 we discuss the validation of the model and provide suggestions for future work. We conclude in Section 9 by stressing the advantages of this approach and highlighting the importance of unifying methods of diverse disciplines in order to design and manage socio-technical systems.

2. Principal-agent problem

2.1. Basic formulation

In game theory (Fudenberg & Tirole, 1991; Leyton-Brown & Shoham, 2008; Rasmusen, 2006), a principal-agent (PA) problem (Laffont & Martimort, 2009) is one in which an uninformed player (the principal) delegates a task to an informed player (the agent) in exchange for a wage. PA models in general are presented in two versions: adverse selection and moral hazard. In this paper, we will focus on the moral hazard problem (Dutta & Radner, 1994). An example of the application of this approach to the particular case of Build-Operate-Transfer (BOT) contracts in public works is presented in Auriol and Picard (2013).

Let's consider the PA problem through the following optimization problem:

$$\max_{w(\cdot)} \mathbb{E}[u_P(q(\tilde{e},\theta), w(q(\tilde{e},\theta)))] \tag{1}$$

subject to

$$\tilde{e} = \arg\max_{e} \mathbb{E}[u_A(e, w(q(e,\theta)))] \tag{2}$$

$$\bar{u} \leq \mathbb{E}[u_A(\tilde{e}, w(q(\tilde{e},\theta)))] \tag{3}$$

where q is the output produced, $w(q)$ is the wage function, e is agent's effort, θ is a random variable chosen by Nature, u_A is the agent's utility and u_P is the principal's utility. The restriction shown in Eq. 2 is called incentive compatibility constraint. It ensures that the agent voluntarily selects his effort for a given contract. The inequality in Eq. 3, called the participation constraint, ensures that the agent prefers the contract to alternative activities that would provide him with a reservation utility $\bar{u}$.

The issue of moral hazard appears because the output is random; i.e., q is the combination of the agent's effort e and the realization of a random variable θ (determined by random exogenous circumstances); therefore, the production level is only a noisy signal of the agent's effort. Because of the stochasticity of the output, it is impossible to directly condition the agent's rewards to the effort he has chosen. In spite of this difficulty, the principal would like to design a wage function $w(\cdot)$ that maximizes her expected utility while acknowledging that the agent will also maximize his own. The PA problem is a bi-level optimization problem (Cecchini, Ecker, Kupferschmid, & Leitch, 2013; Colson, Marcotte, & Savard, 2007) because one of the restrictions (the incentive compatibility constraint, where the agent maximizes his own utility) appears defined as a lower-level optimization problem within the upper-level optimization problem of maximizing the principal's utility.

Analyses of the principal-agent problem in economic literature mostly address canonical static versions such as the one previously described. Other specialized approaches that recognize the problem as a dynamic interaction are less common and fairly recent. Some examples are the applications of performance-based incentives in a dynamical principal-agent model by Plambeck and Zenios (2000), the dynamic moral hazard problem with infinitely repeated actions described by Bolton and Dewatripont (2005) and the continuous-time models by Sannikov (2008) and Cvitanic and Zhang (2012).

2.2. Limitations of the PA model

In general, PA models with moral hazard have two shortcomings at representing the problem of infrastructure procurement; one related to *aggregation* and the other with the *assumption on the knowledge of output*.

Aggregation problem. One of our goals is to model the effects of the players strategies on the dynamic behavior of the infrastructure system. For this reason, it is necessary for us to describe the actions of players as sequential events arranged along a time dimension. Most PA models, however, do not take this approach, but rather, use aggregate variables of effort e and output q, while denoting their relationship as a functional form $q(e)$. Certainly, this simplification is useful for some circumstances and there can be a correspondence between prescriptions of PA models and management problems in firms (Miller, 2008). However, is it not possible to fully represent certain systems (Page, 2012) using aggregation. In our context, the aggregation of discrete inter-temporal actions of players into final variables cannot be used to study the infrastructure's dynamics in which we are interested; we need to model the PA game as a dynamic interaction.

Assumption on knowledge of output. The second inconvenience of the PA models is their assumption that the output is automatically known to the principal just after the effort is exerted. In our problem, the principal is by default ignorant of the state of the infrastructure system (i.e., the equivalent of the output in the PA models); she must actively monitor this output by performing costly inspections to obtain at best a good estimate. Only then she can use a compensation scheme contingent upon the output estimate. Thus, the costly monitoring of the performance must be an essential part of the game.

Considering these two shortcomings, we propose (Section 3) a dynamic model where the compensation scheme could be arranged in such a way that a flat wage is adjusted by penalty fees every time a violation of the performance threshold is discovered, with the hope of discouraging shirking. If we assume that monitoring is realized by a series of discrete and instantaneous inspections, a sequential inspection game arises (see Avenhaus, Stengel, & Zamir, 2002; Dresher, 1962; Ferguson & Melolidakis, 1998; Maschler, 1966). Although the idea of the conflicting inspector-inspectee relationship can be used to describe our problem, we will use different modeling assumptions in order to capture its complexity.

To summarize, the limitations of the PA model presented in this subsection point out that: (1) the players actions and problem variables for the whole of the interaction cannot be meaningfully represented by single real values (e.g., effort e and output q); and that (2) the inspection is a requisite for the principal to estimate the state of the infrastructure. Since, these two specific limitations must be adapted to better represent our problem, a continuous sequential model is proposed in the next section.

3. Continuous sequential model

3.1. Problem overview

The continuous sequential game describes the dynamic interaction between *Principal*, *Agent* and *Nature*; and it is used to evaluate the effect on the performance of an infrastructure system. Then, the *principal* (e.g., government agency) will carry out periodic inspections of the system condition to ensure that it is operating above a pre-specified performance level. She will also impose penalty fees to induce the agent to behave according to her interests. Therefore, the objective of the principal is to maximize her utility by maximizing the expected performance while minimizing inspection costs. On the other hand, the *agent* (e.g., private contractor) is responsible for a voluntary maintenance program according to his internal operational policy; and mandatory interventions if the principal detects that the performance level is below a specified threshold. Note that inspections and voluntary maintenance interventions are proactive actions that do not require triggers to occur. Finally, *nature* encompasses all physical, natural

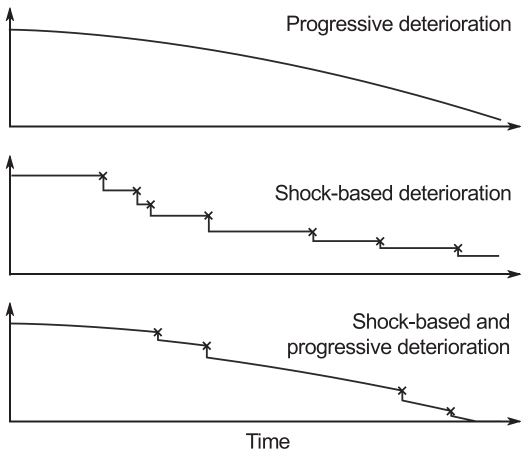

Fig. 1. Types of deterioration. Ordinate axes represent performance level.

Table 1
Available actions to each player and their respective decision variables.

Player	Action	Decision variables
Agent	Voluntary maintenance	Time
		Performance goal
	Mandatory maintenance	Performance goal
Principal	Contract offer	Contract duration
		Payment schedule
		Revenue function
		Performance threshold
	Inspection	Time
	Selection penalty fee	Monetary value
Nature	Shock	Time
		Magnitude of environmental demand

phenomena that affect the system but that are not in control of the agent nor the principal. We introduce the player called 'nature'—a common recourse in game theory—as a participant who does not have preferences and who chooses actions randomly according to some probability distribution instead of strategically. This allows us to include the uncertainty and randomness of the environment into the model. Nature exerts continuous and discrete perturbations to the infrastructure system, which cause progressive or instantaneous degradation (Riascos-Ochoa, Sánchez-Silva, & Akhavan-Tabatabaei, 2014; Riascos-Ochoa, Sánchez-Silva, & Klutke, 2015; Sanchez-Silva et al., 2011). Most physical systems exhibit a combination of the two mechanisms. These two mechanisms and its combined effect are shown in Fig. 1.

In our model all maintenance costs (voluntary and/or mandatory maintenance) must be paid by the agent. For the sake of simplicity, our model does not implement a risk-sharing scheme, since this alone presents several challenges. See further comments in Section 9. The actions of the three players (Principal, Agent and Nature) are summarized in Table 1.

In the sequential continuous game problem, at the beginning (i.e., t_0) the principal selects the parameters of the contract: its duration t_m, the payment schedule h that the principal's promises to pay to the agent, the revenue rate function r_f which is the income rate that the agent receives as a function of the demand of the infrastructure, the performance threshold k, and the penalty policy s_L. Note that this initial move by the principal completely defines the contract.

Then, the possible actions that can be executed by each player are the fundamental building block of the game. Fig. 2 shows the schedule of actions occurring at any time t. What may occur in the game at time $t + dt$ can be constructed simply by connecting the terminal node of the path that was just realized with the root node

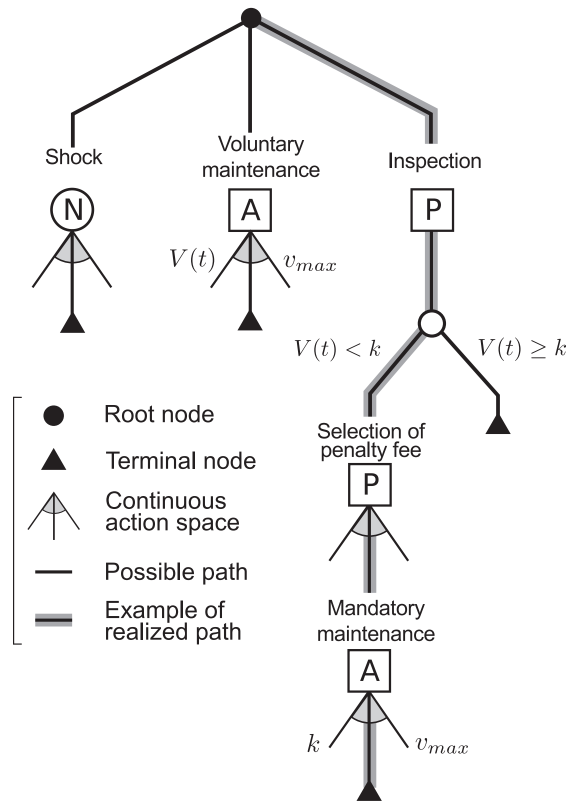

Fig. 2. Unit of sequential game.

of another copy of a tree unit. Then, one can think of each game history as a concatenation of many tree blocks such as those presented in Fig. 2. In Fig. 3 there are two sample paths of the game. Path 1 starts out with the principal selecting a contract where the performance threshold is $k = 60$ while in Path 2, he selects a contract with $k = 40$. Each path is a complete interaction history; i.e., a possible way in which the game can unfold.

3.2. Dynamic interaction

In this sequential game, the ordering of actions is not predetermined and the timing of moves is free for players to decide. Decisions are made based on the following informational setting:

- The value of the performance threshold and the penalty policy is public information.
- The agent knows the stochastic nature of the progressive structural degradation process.
- The agent is aware of inspections once they occur (ex post).

Given these conditions, each player uses a strategy to play the game. A strategy is an algorithm used by a player to decide which actions to perform contingent upon the perceived current state of the world and recalled information. Strategies are immutable throughout the realization of a game. This does not imply a loss of generality, because any strategy composed of a combination of other strategies—even guided by some control algorithm—is itself precisely defined and thus immutable. Players are able to keep historic records and have unlimited recall of observed information and their own executed actions. Whether this ability is used or not depends on the strategy that a player is deploying. The principal's utility depends on his monetary balance and some measure of the cumulative perceived performance of the infrastructure. The agent's utility depends on his monetary balance alone. Players can dynamically perceive their utility as the game evolves.

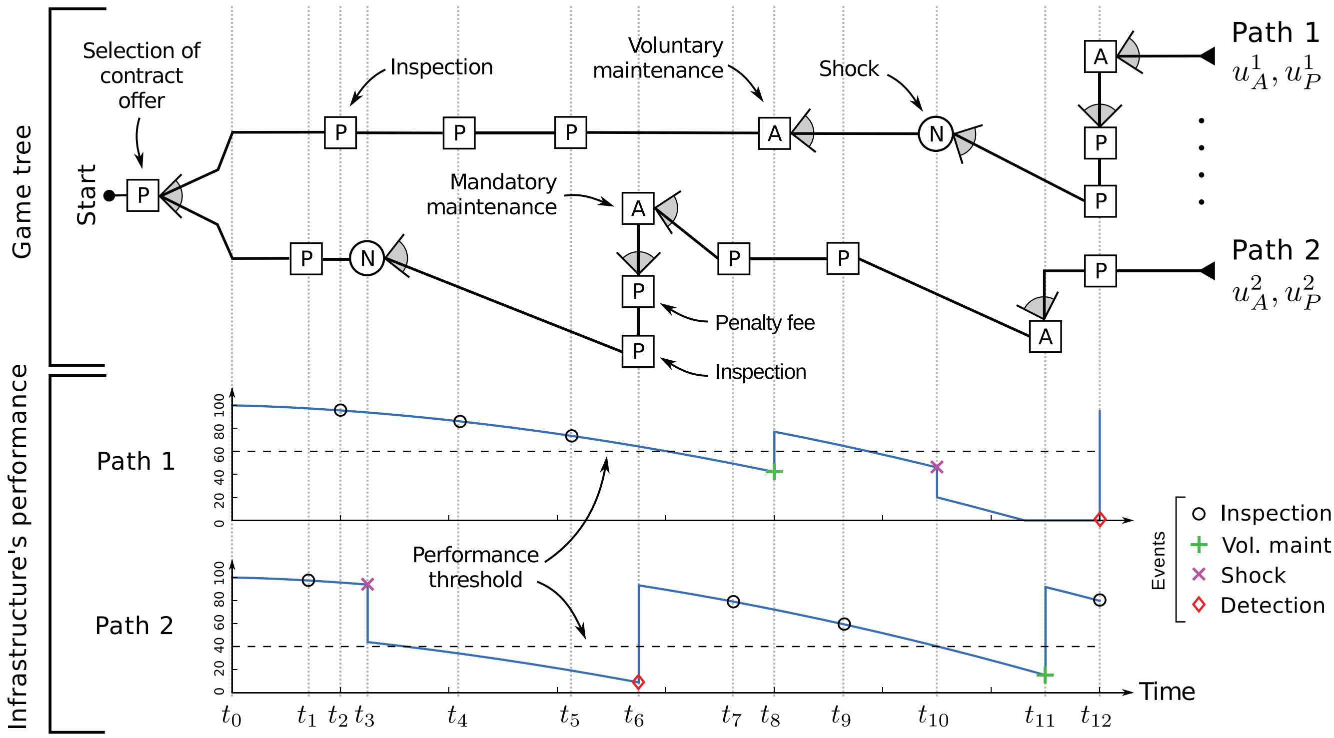

Fig. 3. Representation of two out of infinite possible paths of the sequential game. Top: game tree with players' actions marked as P, A and N for principal, agent and nature, respectively. Bottom: the performance history of an infrastructure system under a deteriorating environment.

The method of backward induction can be used to find a subgame perfect Nash equilibria for finite sequential games with perfect information. However, the proposed game does not meet these conditions since it is an infinite game of imperfect information. It is infinite for two reasons. The first is the potentially uncountably infinite number of nodes derived from the continuity of the time dimension. The second is that this game has at least one continuum action set, thus from a decision point of such action, uncountably infinite paths branch off. Because the conditions mentioned are not met, the algorithm of backward induction is not well defined (Fudenberg & Tirole, 1991, p. 91) and is therefore not applicable to our problem.

4. System dynamics

Let's now define more precisely the relationship between all different actors and the mechanism by which infrastructure evolves (Fig. 3). Then, we will frame the concepts of game theory presented so far, within the context of *System Dynamics* (SD) (Forrester, 1973, 2013). SD is a mathematical methodology that simulates the behavior of a complex system by identifying its parts and the connections between those parts in the form of relationships of dependence. The system dynamics of our model can be described by the stock-flow diagram presented in Fig. 4. It includes all components of the model and the flow of information. In the following we will describe only the most important aspects.

Let's first define the stocks in this model, depicted as tanks in Fig. 4; these are: (1) the performance of the infrastructure; (2) the agent's monetary balance; and (3) the principal's monetary balance. Furthermore, the variations on the *performance measure* of infrastructure is increased by flows of continuous and discrete maintenance works and is decreased by flows of progressive (continuous) and shock-based (instantaneous) degradation. Note that nature exerts its influence in the form of progressive and instantaneous environmental forces. These forces are received by the infrastructure system and are translated into progressive and instantaneous degradations through a response function. Such response function also depends on the current level of performance and the demand of users which could potentially have a damaging or repairing effect.

The *agent's balance* is increased by the payment schedule promised by the principal and the revenue caused by the operation of the infrastructure. It is decreased by the maintenance costs and penalty fees. The revenue rate depends on the users' demand, which in turn depends on the infrastructure's performance. Maintenance costs are determined through the actions that the voluntary maintenance and the mandatory maintenance strategies implement. Penalty fees are determined by the penalty strategy. On the other hand, the *principal's balance* is increased by the penalty fees and it is decreased by the payment schedule agreed in the contract and by the cost of inspections. The values of the payment schedule are transferred to the agent. When the inspection strategy dictates an inspection action, its cost is instantaneously subtracted from the principal's balance and a record of the current performance is observed and stored by the principal. When an inspection is executed, a level of compliance is calculated based on the perceived performance from an inspection and the performance threshold.

A cloud in Fig. 4 represents a source or sink of a flow; they mark the boundary of the system. We do not keep track of anything that is beyond the clouds. For instance, while the monetary value coming out of the agent's balance through the maintenance cost flow may actually be received by a subcontractor who is hired to perform a specific reparation, it is a process that is outside the scope of the analysis. Furthermore, small circles in Fig. 4 represent calculated elements or simple parameters. Rhombi represent strategies. They receive some input and produce signals to other components with information about dictated actions. Since the information input of strategies depend completely upon their particular internal structure, no dependency is drawn as a definitive

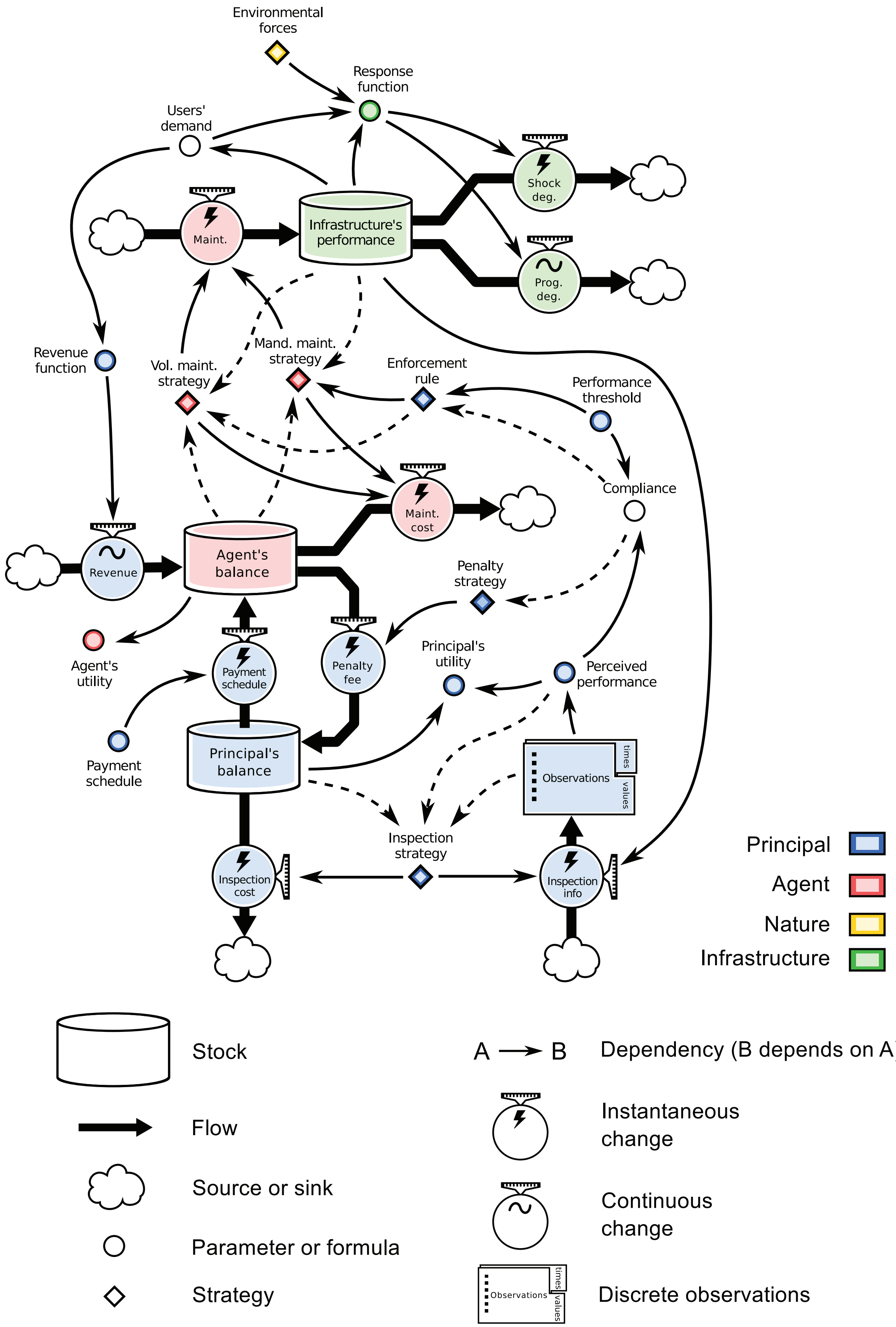

Fig. 4. Conceptual stock-flow diagram of dynamic PA game. Each element is identified with the color of the entity that owns it or controls it.

relationship. Instead, the possibility of such dependencies is drawn with dotted lines. Although the existence of these relationships seem intuitive, they do not imply necessary dependence but rather the possibility to information access if needed. As an illustration of this, suppose the agent chose a strategy for both mandatory and voluntary maintenance that at fixed time intervals would increase the performance by a fixed amount. The execution of such strategies would not require the agent to know the performance previous to the intervention. In contrast, if the voluntary maintenance strategy dictated that each intervention would bring the infrastructure to a specific performance, then the knowledge of its previous value would indeed be required.

When the suggested dependencies with dotted lines are considered, interesting feedback loops appear. Take for example the voluntary maintenance strategy. It sends an information signal of a maintenance action to the maintenance flow, which increases the infrastructure's performance stock. In turn the infrastructure's performance affects the next action produced by the voluntary maintenance strategy. This feedback loop is very simple and its effect may be instantaneous, but others run across more components and may exhibit delays. For instance, the action produced by the voluntary maintenance strategy controls the maintenance flow, which increases the infrastructure's performance. The value of performance may be observed and registered by the principal, who will use it to estimate the level of compliance based on a chosen performance threshold. Such compliance is an input of the penalty strategy, which sends a signal to the penalty fee flow, which in turn affects the agent's balance. Finally, a change in the agent's balance will affect the next action dictated by the voluntary maintenance strategy itself, thus completing the loop.

In summary, the dynamic system model proposed inhere provides a coherent integration of the parts that compose our PA problem by including the following aspects:

- *Original PA problem*: the existence of two players (principal and agent) with information constraints (asymmetry included), conflicting goals and a promised wage or payment schedule agreed in a contract between the two parties.
- *Natural environment*: a fictitious player called Nature whose actions are uncertain for the principal and the agent.
- *Infrastructure system*: the continuous and discrete dynamics of an infrastructure system upon which the principal, agent and nature operate.
- *Inspection game*: the necessary costly inspections that the principal uses to learn information about the infrastructure and estimate agent's actions. The definition of agent's legal and illegal actions according to a specified minimum performance threshold. Also the inclusion of threats in the contract in the form of penalty fees to be imposed on the agent if a violation of the minimum threshold is detected during an inspection.
- *Players' actions*: the definition of specific actions by which the players interact. For the principal, they are the selection of the contract (e.g., payment schedule, performance threshold), the execution of inspections and the imposition of penalty fees. For the agent, they are the execution of a voluntary maintenance and the execution of a mandatory maintenance. For nature, they are the imposition of discrete shocks and continuous deteriorating forces.
- *Exogenous parameters*: the parameters that should be chosen by the modeler to match a particular instance of the problem, such as the response function of the infrastructure system, the character of the users' demand and the revenue earned by the agent as a result of the operation of the infrastructure system.

5. Mathematical formulation of the hybrid model

5.1. Hybrid system dynamics

The combination of continuous and discrete behavior in a system is denoted with the term *hybrid* (Goebel, Sanfelice, & Teel, 2012). The theory of hybrid system dynamics have been used extensively to model mechanical and electrical systems (Goebel et al., 2012), but can be easily extended to other systems. A hybrid system can move throughout its state space both in a continuous and instantaneous manner. The continuous evolution of the system is given by a differential equation (or set of differential equations) called a flow map. On the other hand, the instantaneous evolution is described by a recurrence relation, called a jump map. Furthermore, there are certain conditions that determine whether the system flows or jumps at each particular instant.

It is characteristic of our problem that some aspects are better described as continuous and smooth and others as discrete and sudden. For example, the progressive deterioration of a physical system can be modeled as a continuous process, while the action of a player is better represented as a discrete event that causes instantaneous change. We argue that the framework of hybrid systems is a useful and natural analogy that encompasses the perspective of game theory (interaction of players through execution of instantaneous actions) and system dynamics (the smooth evolution of variables described by differential equations).

Hybrid models are parameterized by a set $E \subset \{\mathbb{R}_{\geq 0} \times \mathbb{N}\}$, where the vector $(t, j) \in E$, defines the time t and the order of discrete jumps j in the system state (Fig. 5). Note that the hybrid time domain allows the possibility of more than one jump occurring at the same value of continuous ordinary time t while capturing the order of its occurrence. Also, it allows to unambiguously refer to the state of the system that exists just before or after an instantaneous event.

As an illustrative example, let us suppose that the state of an infrastructure system is defined by the scalar variable $x = V \in X$, denoting its performance. When the performance of the infrastructure is plotted (see right plot in Fig. 5), the time intervals where the performance value is continuous and smooth are separated by sudden jumps that occur as a result of shocks and maintenance actions. The regions where the continuous evolution occurs are governed by a differential equation. On the other hand, sudden jumps are governed by a recurrence relation, which in the context of this problem is an abstraction for the collective action taken by the players when they are confronted with some state of the world.

Based on this representation, let us propose that the game between Principal, Agent and Nature is a hybrid system, where its state is defined by a vector $x \in X$, where X is the system state space. This notion will be expanded further in Section 5.7.

5.2. Actions

The three players *Agent*, *Principal* and *Nature* will be denoted Player 1, 2 and 3 respectively. By including Nature as a third player, the game has three proactive actions: inspection (principal), shock—instantaneous degradation—(nature; e.g., earthquake) and voluntary maintenance (agent). Between actions, the system may or may not degrade continuously. There are two reactive actions that occur when a violation is detected: mandatory maintenance and the selection of an enforced penalty fee.

The game has four states defined by the set $\Gamma = \{0, 1, 2, 3\}$ whose elements represent:

$\gamma = 0$: initial state; the principal offers the contract to the agent and he accepts it.

$\gamma = 1$: all players select a proactive action.

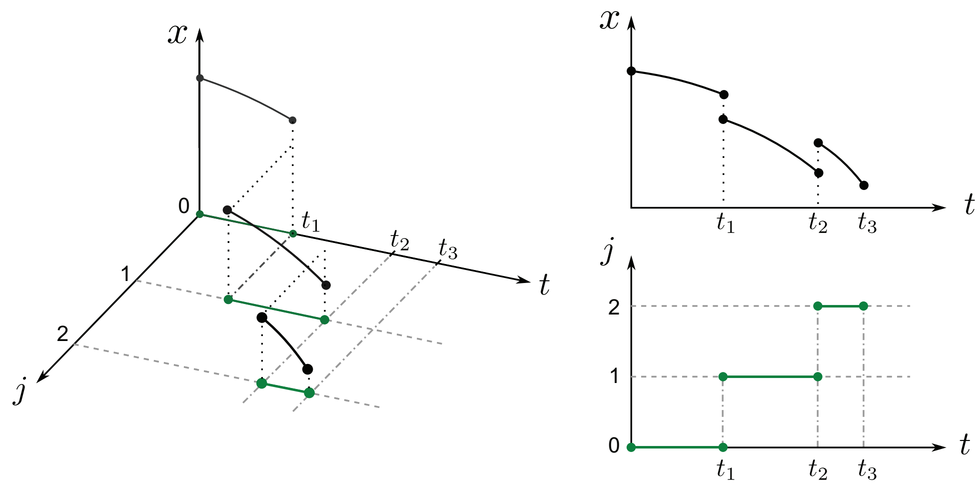

Fig. 5. Representation of performance V parameterized in hybrid time (t, j).

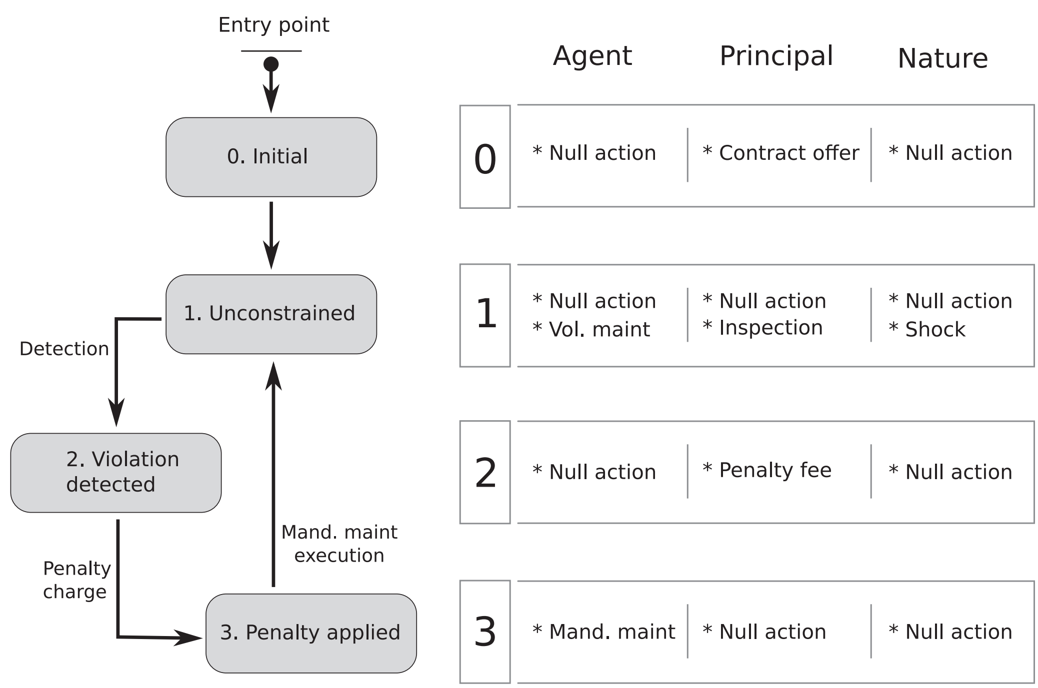

Fig. 6. State chart of the game. Each state shows the correspondent available actions for players.

$\gamma = 2$: a detection has occurred and the principal selects a penalty fee.

$\gamma = 3$: the penalty has been charged and the agent must perform a mandatory maintenance.

Fig. 6 shows the possible states of the game, the actions available to each actor and the events that trigger the transition between states. We refer to the space of all possible instances of a particular action as an *action set*. Let us now formally describe the actions shown in Fig. 6 as action sets for each player as a function of time and the state of the game. The variable $\gamma \in \Gamma$ indicates the current state of the game.

- *Agent's action set*: The complete action set for the agent (player 1) at (t, j) is

$$A_1^{(t,j)} = \begin{cases} O & \gamma = 0 \\ \{O \cup \Lambda^{(t,j)}\} & \gamma = 1 \\ O & \gamma = 2 \\ M^{(t,j)} & \gamma = 3 \end{cases} \tag{4}$$

where O is the null action set, $\Lambda^{(t,j)}$ is the voluntary maintenance action set and $M^{(t,j)}$ is the mandatory maintenance action set. A unique maintenance action is defined by a vector (v_i, v_f), denoting the performance before and after its execution. These vectors constitute the voluntary maintenance action set $\Lambda^{(t,j)}$ and the mandatory maintenance action set $M^{(t,j)}$.

- *Principal's action set*: the complete action set for the principal (player 2) is

$$A_2^{(t,j)} = \begin{cases} C & \gamma = 0 \\ \{O \cup \mathcal{I}\} & \gamma = 1 \\ L & \gamma = 2 \\ O & \gamma = 3 \end{cases} \tag{5}$$

where C is the contract offer action set, $\mathcal{I}$ is the inspection action set and L is the penalty fee action set. The inspection action set $\mathcal{I}$ contains only an element which represents the execution of an inspection. The values contained in the penalty fee action set L represent the possible monetary penalty that the principal imposes on the agent after a violation is detected.

Table 2
Spaces and action sets.

Spaces	
$\Upsilon = \{v \in \mathbb{R} \mid v_{min} \le v \le v_{max}\}$	Performance space of infrastructure system
$\Upsilon_\omega = \{(v_i, v_f) \in \Upsilon^2 \mid v_i < v_f\}$	Maintenance space
Action sets	
$O = \{\tilde{o}\}$	Null action set
C	Contract offer action set
$\Lambda^{(t,j)} = \{(v_i, v_f) \in \Upsilon_\omega \mid v_i = V^{(t,j)}\}$	Voluntary maintenance action set
$M^{(t,j)} = \{(v_i, v_f) \in \Upsilon_\omega \mid v_i = V^{(t,j)} \wedge v_f \ge k\}$	Mandatory maintenance action set
$\mathcal{I} = \{\iota\}$	Inspection action set
$L \subset \mathbb{R}_{\ge 0}$	Penalty fee action set
$\Xi \subset \mathbb{R}_{\ge 0}$	Shock action set

A contract offer is a vector $(t_m, h, r_f, k, s_L) \in C$, where $t_m \in \mathbb{R}_{\ge 0}$ is the contract duration, $h : \mathbb{R}_{\ge 0} \mapsto \mathbb{R}_{\ge 0}$ is the payment schedule that relates time with the principal's contributions, $r_f : \mathbb{R}_{\ge 0} \mapsto \mathbb{R}_{\ge 0}$ is the revenue rate function (see Eq. 8), $k \in \Upsilon$ is the performance threshold and s_L is the penalty fee strategy to which the principal commits to use in case of detections.

- *Nature's action set*:

$$A_3^{(t,j)} = \begin{cases} O & \gamma = 0 \\ \{O \cup \Xi\} & \gamma = 1 \\ O & \gamma = 2 \\ O & \gamma = 3 \end{cases} \tag{6}$$

where Ξ is the shock action set. The values contained in the shock action set Ξ represent the magnitude of instantaneous environmental force. Shocks may be used to model earthquakes, floods, fires or other catastrophic events that can be considered as sudden events.

The details of other sets that make up $A_1^{(t,j)}$, $A_2^{(t,j)}$ and $A_3^{(t,j)}$ are formally defined in Table 2. The null action set is included to take into account that this game allows only one proactive action to be executed at a time, therefore whenever a player executes an action, the others must necessarily be forced to choose $\tilde{o}$ for the time instant. Also, all three players may simultaneously choose to do nothing at a time t, in which case, the continuous environmental force will cause the infrastructure to deteriorate. With the inclusion of a null action available to the players in the unconstrained state $\gamma = 1$, it can be asserted that they all make a choice of action continuously.

The information setting is central to the definition of a game. The information accessible to the ith player at time (t, j) is described by the variable $\chi_i^{(t,j)}$. In particular, the variables χ_i will provide the considerations of information accessibility described in Section 3 where the sequential inspection game was defined. We will assume they provide the ith agent with information previously recorded by him as well as information signals that are currently perceived.

5.3. Functions

As seen in Fig. 4, the rate of change of the stocks are affected by auxiliary functions connecting various components of the system. Here, we present a description of each function.

- *Demand*: the usage level of the infrastructure per time unit is expressed in the model as a user demand function

$$d = d_f(V, t) \in \mathbb{R}_{\ge 0} \tag{7}$$

which may represent, for example, the number of concurrent users in the case of a public transportation system or the specific stress exerted on the infrastructure; for instance, the number of Equivalent Standard Axial Loads (ESAL) per time unit in the case of a pavement structure or the power demand in Megawatts of an electrical distribution network.

- *Revenue rate*: the revenue function

$$r = r_f(d) \in \mathbb{R}_{\ge 0} \tag{8}$$

is the continuous income stream per time unit that the agent receives as a result of the operation of the infrastructure system. In the case where users of the infrastructure directly pay the agent for the service provided (e.g., a toll road) the revenue rate is determined by the instantaneous demand from users.

- *Environmental forces*: the environment naturally imposes disturbances that can directly change the state of the infrastructure. The term environmental forces is used to refer to such disturbances. The continuous environmental force

$$\tilde{f} = f_c(t) \in \mathbb{R}_{\ge 0} \tag{9}$$

may represent eroding factors like rain and wind in geotechnical structures or sea waves in coastal structures. However, its nature does not necessarily imply mechanical stress. It can also be used to model the presence of chemical corrosives like chloride that deteriorates reinforced concrete structures or changes in the water table that may affect the reliability of a foundation.

- *Infrastructure response*: an infrastructure system is assumed to have the property of a known response function to environmental forces (both continuous and discrete) and the demand level (i.e., usage level). The response function can be separated into a continuous response function

$$\delta_c = r_c(\tilde{f}, d, V, t) \in \mathbb{R}_{\ge 0} \tag{10}$$

which determines the rate of progressive deterioration and a discrete response function

$$\Delta_s = r_d(\hat{f}, V, t) \in \mathbb{R}_{\ge 0} \tag{11}$$

that produces the shock-based deterioration, where $\hat{f} \in \Xi$.

- *Maintenance cost*: the maintenance cost function

$$\psi : \Upsilon_\omega \mapsto \mathbb{R}_{>0} \tag{12}$$

maps a maintenance intervention to its respective cost. This function describes the level of efficiency of the agent's operations. Additionally, we define the general form of the players utility functions:

- *Utility functions*: the agent's utility is given by the function

$$u_A = u_1(b_A) \in \mathbb{R} \tag{13}$$

where b_A is the agent's monetary balance and $\partial u_1 / \partial b_A > 0$. The principal's utility is

$$u_P = u_2(b_P, \tilde{V}) \in \mathbb{R} \tag{14}$$

where b_P is the principal's monetary balance, $\tilde{V}$ is a measure (e.g., the mean value) derived from the performance observations.

Table 3
Parameters in the model.

Related entity	Symbol	Parameter
Problem	d_f	Demand function
Infrastructure	v_{min}	Null performance
	v_{max}	Maximum performance
	v_0	Initial performance
	r_c	Continuous response function
	r_d	Discrete response function
Nature	f_c	Continuous environmental force
	s_3	Nature's strategy (shocks)
Agent	ψ	Maintenance cost function
	b_A^0	Agent's initial balance
	u_A	Agent's utility function
Principal	c_ι	Cost single inspection
	u_P	Principal's utility function

5.4. Parameters

The parameters of the model are summarized in Table 3. These are elements that are associated with a particular instance of the problem. Five of them are values: v_{min} and v_{max} are the limits of the performance space, v_0 is the performance of the infrastructure at the beginning of the game, b_A^0 is the initial agent's balance which is normally composed of the initial payment received from the principal (if any) minus the initial investment or construction cost, and c_ι is the cost that the principal incurs every time she inspects the infrastructure. Seven other parameters are functions, which were defined in the previous subsection.

As we mentioned before, nature does not have preferences and does not act strategically. Therefore, the parameter s_3 is simply an algorithm that dictates actions from the action space A_3 (see Eq. 6). The (strategy) algorithm is chosen by the modeler to resemble environmental pressures from a particular problem instance. The next subsection specifies what a strategy is and how it relates to actions.

5.5. Strategy sets

At every time instant, each ith player performs an action $a_i \in A_i(t, j)$ dictated by some strategy s_i. A strategy for the ith player is the relation

$$a_i = s_i(\chi_i^{(t,j)}, \gamma) \in A_i^{(t,j)} \tag{15}$$

This notation implies that the strategy s_i produces actions according to the action space that is available to player i at time (t, j) as a function of the state of the game γ. An agent's strategy s_1 encapsulates a voluntary maintenance strategy and a mandatory maintenance strategy. Similarly, a principal's strategy s_2 encapsulates the inspection strategy and the penalty fee strategy. In the case of nature, s_3 simply contains the shock strategy. In this particular case, the use of the term strategy does not mean that we assume nature has intentions behind the exertion of a shock; it does not imply rationality or agency. Rather it is used to denote the process by which shock actions are produced.

The realization of one game has a unique combination of strategies, also called strategy profile denoted as $\mathbf{s} = (s_1, s_2, s_3)$. If we define an information vector $\boldsymbol{\chi} = (\chi_1, \chi_2, \chi_3)$, then a strategy profile

$$\mathbf{a} = \mathbf{s}(\boldsymbol{\chi}) \tag{16}$$

transforms an information vector into an action profile $\mathbf{a} = (a_1, a_2, a_3)$ which is the joint selection of actions at a particular time in the game. The strategy set S_i of the ith player contains all his available strategies. In order to play the game, the ith player selects a specific strategy out of his strategy set $s_i \in S_i$. All possible combinations of players strategies are included in the strategy space

$$S = \{S_1 \times S_2 \times S_3\} \tag{17}$$

A strategy profile is therefore a point within the strategy space, so that $\mathbf{s} \in S$.

5.6. State space and transitions

If X is the state space of the game, then it is defined as $X = \{X_1 \times X_2 \times \ldots \times X_n\}$, where n is the number of variables that compose the state of the game and X_n is the state space of the nth variable. If $x \in X$ is a particular state, in our problem it is composed of at least the variables

$$x = (V, b_A, b_P, \gamma, \ldots) \tag{18}$$

thus, the state space of our problem is $X = \{\Upsilon \times \mathbb{R} \times \mathbb{R} \times \Gamma \times \ldots\}$. Ellipses are used because there is a large number of variables that we could track in the model which are present in any computational implementation of the problem. However, for the scope of this work, we are only interested in those explicitly written in Eq. 18. It is possible now to interpret each path in the game tree in Fig. 3 as a possible trajectory of the state vector x within the state space X, whose motion was dictated by the aggregation of the players' strategies and the naturally occurring phenomena of the physical infrastructure system. The process where the players select and perform their actions, depicted in Figs. 2 and 6 are the equivalent of a jump map

$$x^{(t,j+1)} = \tau(x^{(t,j)}, \chi^{(t,j)}) \tag{19}$$

while the flow map can be represented by the equations

$$\frac{dV}{dt} = r_f(d_f(V,t)) \tag{20}$$

$$\frac{db_A}{dt} = r_c(f_c(t), d_f(V,t), V, t) \tag{21}$$

$$\frac{dV}{db_P} = 0 \tag{22}$$

$$\frac{dV}{d\gamma} = 0 \tag{23}$$

which both depend on V and t. A complete summary of the notation of the mathematical formulation presented in this section is shown in Appendix A.

5.7. Game execution

The exact process by which the hybrid system evolves—both discretely (jump map) and continuously (flow map)—is presented in the Algorithm 1 and Algorithm 2 in Appendix B. All variables, strategy and action spaces, functions and parameters are included in a detailed process that describes the actual execution of a game realization.

6. Implementing the game

As the formulation for the game progressively grows in complexity, it becomes more difficult to fit its structure into the basic models provided by game theory. Then, in order to recreate the game as formulated in the previous section without incurring in further simplifications, we developed an hybrid simulation model that combines System Dynamics (SD) and Agent-Based Modeling (ABM).

6.1. Agent-based model

Agent-based modeling is a simulation method composed of agents that interact within a given environment (Gilbert, 2008). Each agent can be autonomous and adaptive. Their aggregate interaction dictates the evolution of the whole system, which often exhibits complexity out of very basic rules of individual behavior. For this reason, ABMs are useful to model the properties of complex systems (Railsback & Grimm, 2011). Agent-based modeling is also related to the fields of discrete and continuous dynamical systems, multi-agent systems (Shoham & Leyton-Brown, 2008) and game theory (Axelrod, 1984). ABM has also influenced the social sciences (Epstein, 1999). The advent of powerful computers combined with the ABM paradigm has provided a framework to better understand complex emergent phenomena in social systems composed of individuals (i.e., agents) by simulating the evolution of their interactions (Helbing, 2012). The remarkable advantage of ABMs is that besides simulating socio-economic interactions, they could simultaneously emulate a realistic description of physical interactions between agents. The addition of the latter feature is not yet widely used, and we have found only in Sanford Bernhardt and McNeil (Sanford Bernhardt & McNeil, 2008) an instance with the mentioned feature being specifically targeted at modeling the life-cycle of civil infrastructure with the perspective of a socio-technical system.

The proposed model combines system dynamics with agent-based modeling; an overview on the design of hybrid AB-SD simulation models can be found in Swinerd (Swinerd & McNaught, 2012). The selection of a hybrid SD-AB simulation corresponds nicely with the properties of hybrid system dynamics presented in Section 5 to formally describe the game. They both share the ability to represent continuous and discrete processes.

In this paper the basic principal-agent game is extended and modeled as a sequential game between autonomous players and their environment. All moves are computed by a strategy that each player selects at the beginning of the game. The simulation is a time-dependent game that in its finished state produces an interaction history from which the aggregate utility for each player can be calculated. The realization of a game is therefore a transformation of the players strategies and the problem parameters ϕ into the utilities

$$(u_A, u_P) = \mathcal{G}(s_A, s_P, \phi) \tag{24}$$

where $s_A \in S_1$, $s_P \in S_2$. Strategies for inspection, maintenance and penalty policy selection can be added to the strategy set of a player (i.e., S_i) and be selected to evaluate $\mathcal{G}$. The result of such game will show the emergent effect of these strategies throughout the life-cycle of the infrastructure.

6.2. Entities

The agent-oriented computational paradigm (Shehory & Sturm, 2014) offers valuable guidance on the design of agent-based simulation models. The hybrid simulation model was implemented in MATLAB as an object-oriented program where the problem is represented by objects with attributes, methods and associations (Booch, 2007). The entire model as presented in this paper is available at the public repository https://github.com/davpaez/contract-design. We will not explain the details of the implementation, since many configurations of the actual computer program could replicate the model described in Section 5. The overview of the most important components of the object-oriented program and their relationships are shown in Fig. 7 using UML (Unified Modeling Language). It is observed in this figure that the classes Principal, Agent, Nature and Contract have strategies that produce the action profile at every time instant. Strategies are composed of objects called decision rules. The use of a decision rule is to compute and return one or more decision variables of an action. Thus, a strategy responsible for producing an action with m decision variables may contain up to m decision rules that collectively compute the value of all decision variables. Decision rules are therefore the building blocks of the player's behavior, strategies are collections of decision rules that compute the decision variables required to define an action.

6.3. Simulation

When the system is within the flow set, the model uses a numeric method for solving ordinary differential equations. This behavior in the model is typical of a SD simulation. When the system is within the jump set, the model behaves as an ABM simulation (see Fig. 4). In contrast with the mathematical formulation in the definition of the hybrid system, instead of continuously asking players to produce an action profile, the Realization class arranges an iterative process where it asks players to submit the next action they wished to perform, while ignoring null actions. Only the earliest action is allowed to be executed. If the current time of the system is less than the time of the allowed action, the system evolves according to the flow map until such time is reached. Then the action is executed and players are asked to submit their next action once again. In this manner, the process is repeated until the duration of the contract is reached. This is in practice identical to the Algorithm 1.

Since the model proposed aims to represents granular characteristics, it also needs a set of parameters. These parameters are related to the specific problem instance that the modeler wants to simulate. They broadly refer to properties of the problem, the infrastructure system, the contract, nature, the principal and the agent. The input data requirements were summarized in Table 3. Having provided the input data requirements, it is the player's strategies themselves which remain to be specified in order to run a game realization. By making use of the decision rule/strategy hierarchy described earlier, the modeler can devise various strategies and assign them to players. By doing so, the modeler forms a strategy profile, which combined with the parameters, are used to evaluate the game $\mathcal{G}(s, \phi)$. The use of the simulation model—with a government agency in the role of user/modeler—is summarized in Fig. 8. Controlled parameters are those that the government agency (i.e., principal) can voluntarily select, such as the kind of infrastructure system and the meaning and measure of the performance level (e.g., v_{min} and v_{max}). Uncontrolled parameters are a condition of the real-world system, such as the distribution of shocks exerted by nature s_3, or the demand function d_f.

7. Numerical experiments

We have covered in Section 5 the theoretical structure of the problem and in Section 6 the arrangement of its implementation as a hybrid AB-SD model. This section will present numerical experiments for a specific problem instance. We will characterize the following problem by specifying the parameters listed in Table 3 and the strategies that each player deploys.

7.1. Problem description: Construction and operation of a highway

Consider a PPP whose goal is the construction and maintenance of an interurban highway. The principal is a government agency who is in charge of the local transportation network. The agent is a private firm or consortium of firms with access to credit. The principal offers a BOT contract to the agent. The performance of a road is defined as its ability to serve traffic. Examples of existing road performance measures are the PSI (Present Serviceability

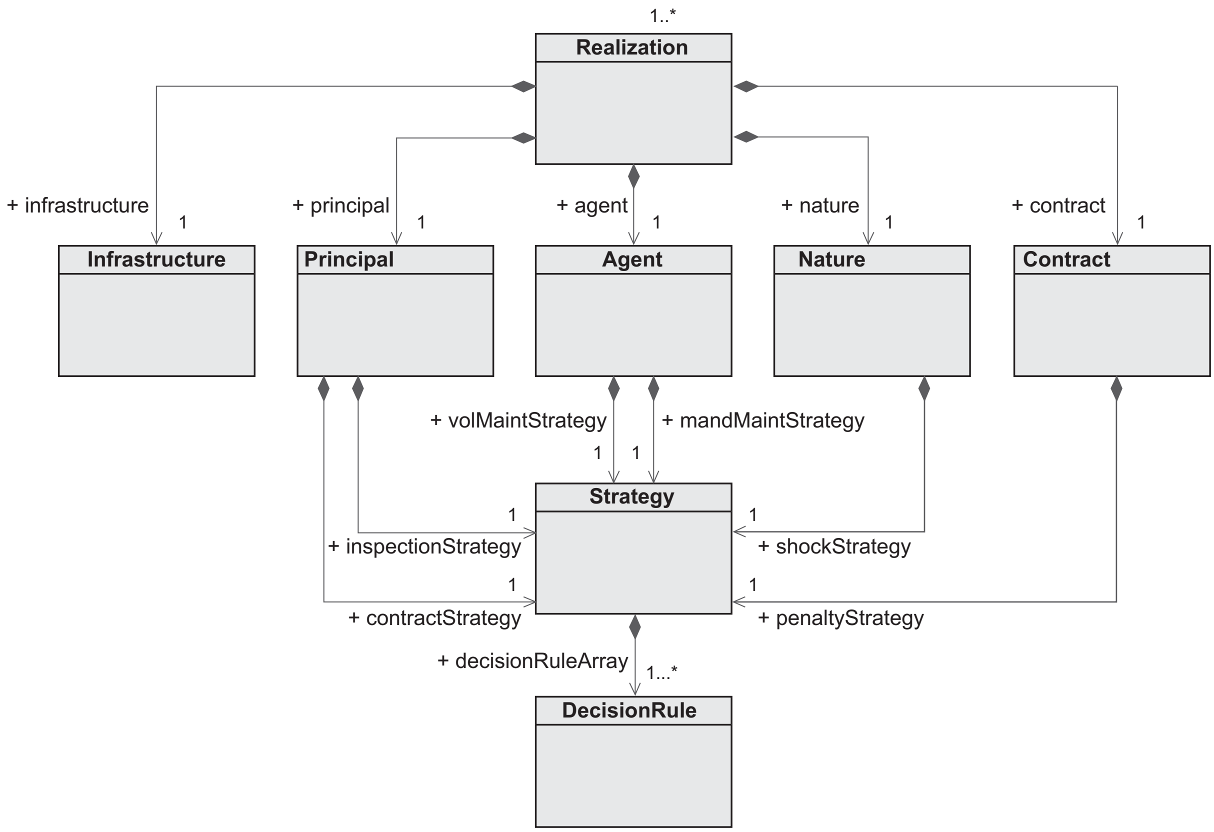

Fig. 7. Simplified class diagram of the ABM, features the most important *association relationships*. Inheritance relationships are not shown.

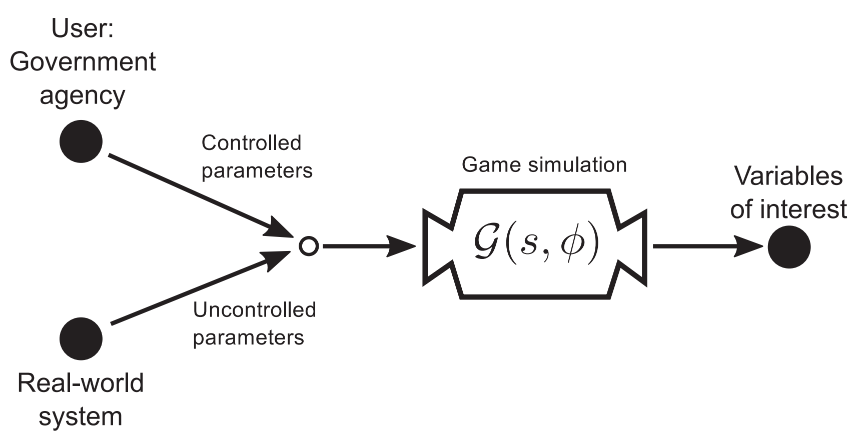

Fig. 8. Use of the simulation model.

Index, a ride quality rating) and the IRI (International Roughness Index, a roughness estimation based on a measured longitudinal road profile) (Sayers, Gillespie, & Paterson, 1986). These measures are very useful in practice for road management and they are related to vehicle operation costs (Chatti & Zaabar, 2012).

Let's now assume the infrastructure performance space Y is bounded by $v_{min} = 0$, $v_{max} = 100$. The initial performance is $v_0 = 100$. The traffic flow is mostly composed of small vehicles which cause imperceptible damage to the pavement structure. The continuous response function in all experiments will be independent of demand and continuous environmental forces.

We make the assumption that the agent is risk neutral and that his utility is exactly equal to the balance of the monetary values he perceives. We can define such function as

$$u_A = b_P = b_A^0 + \sum h - \rho - \mu \tag{25}$$

where Σh is the total of contributions received in the payment schedule, ρ represents the penalties imposed by the principal and μ the total maintenance cost. The agent's initial balance b_A^0 is equal to the construction cost $cc = \$875$. The model assumes cc to be independent of the players' strategies. The components h, ρ and μ do depend on the players' strategies.

The maintenance cost function is linear with respect to the change in performance; i.e.,

$$\psi = \frac{V_f - V}{v_{max} - v_{min}} \cdot \epsilon \cdot cc + \$4 \tag{26}$$

where V_f is the final performance after the maintenance is completed and $\epsilon = 0.2$ is the fraction of the construction cost needed to perform a perfect maintenance (a maintenance whose final performance is v_{max}) starting with a null performance.

Table 4
Results from numerical experiments. Results from experiments 5 and 6 are expected values.

Experiment	Features	u_A	u_P	b_P	$\hat{V}$	$\bar{V}$
1	Non adaptive strategies	468.95	0.92	−410.5	93.42	92.77
2	Adaptive vol. maint. strategy	574.47	0.89	−410.5	89.95	72.56
3	Demand function	668.1	0.93	−410.5	93.65	70.89
4	Reduced inspection interval	927.1	0.93	−421.0	95.03	81.65
5	Random inspections	658.3	0.66	−214.8	84.96	82.00
6	Random shocks	−25.4	0.56	−122.32	83.48	78.82

We define the principal's utility as

$$u_P = \begin{cases} (10\hat{V} - b_P - \sum h)/1000 & b_P \geq -\sum h \\ (10\hat{V} + b_P + \sum h)/1000 & \text{otherwise} \end{cases} \tag{27}$$

where $\bar{V}$ is the perceived mean performance that she estimates by approximating the real degradation path of the infrastructure with linear interpolation between inspection samples. This utility achieves its highest value when $b_P = -\sum h$ and $\hat{V} = v_{max}$. The principal balance is

$$b_P = \rho - \sum h - \sum c_\iota \tag{28}$$

where Σc_ι is the total inspection cost. The cost of a single inspection is $c_\iota = \$1.75$.

7.2. Contract definition

The following is the contract offer. Its duration is 25 years. The government payment schedule h consists of four payments $ [150, 150, 50, 50] at [0, 5, 10, 15] years. The revenue rate function is

$$r_f = fare * d \tag{29}$$

where $fare = 6 \times 10^{-6}$ and d is demand which comes in the form of traffic moving along the road segment, measured as vehicles per year. The performance threshold is $k = 70$. We use a simple penalty policy that always charges $50 for every violation.

7.3. Numerical experiments

Six experiments were carried out to show how the output of the game changes depending on strategies, parameters and functions. The examples introduce changes gradually in order to show their marginal effect. A summary of the features and results of each experiment is shown in Table 4. The perceived mean performance is denoted as $\hat{V}$ and the real mean performance as $\bar{V}$.

Experiment 1. The continuous response function is $r_c = -17.16$. This constant rate causes the infrastructure, if it remains undisturbed, to go from v_{max} to v_{min} in 5.83 years. Additional assumptions include:

1. The demand is a constant rate of $d_f = 12 \times 10^6$ vehicles per year.
2. There are no shocks in this experiment.
3. The principal and agent have non-adaptive strategies:
4. The principal performs inspections at fixed intervals of 4 years and
5. The agent performs perfect maintenance interventions at intervals of 0.85 years.

Because the strategies deployed are non-adaptive, no player would respond to a change in the other player's strategy. For instance, more frequent inspection would have no effect on the agent's or his utility.

Fig. 9 shows the dynamic behavior of some of the variables of interest in an experiment: the performance level along with special markers representing events (these were introduced in Fig. 3), the monetary balance of the principal and the agent, the real mean performance of the infrastructure and its perceived value as estimated by the principal. This representation is useful to visualize the strategies deployed and their combined effect in the interaction process.

Experiment 2. The agent now deploys an adaptive voluntary maintenance strategy. It works by estimating when the next inspection will occur—based on the assumption that they are regularly spaced—and performing a maintenance intervention just before it takes place. This change in strategy allows the agent to perform less maintenance works without being detected when the performance goes below the threshold.

In this case, the agent's utility increases 22.5%. As a result of the agent's strategy, the principal is overestimating the system's performance. He only perceives a slight reduction of 3.7% in $\hat{V}$ whereas $\bar{V}$ shows a 21.8% reduction with respect to experiment 1. The representation of the dynamic output from experiments 2 to 4 is shown in Fig. 10.

Experiment 3. Let's now define the following continuous response function (which replaces the continuous response function used in experiments 1 and 2):

$$r_c = -0.01 - 2.5779 \cdot (v_0 - V)^{0.5652} - 0.6t \tag{30}$$

Note that it only depends on initial performance, current performance and time. This function also causes the infrastructure to completely deteriorate in 5.83 years, but its degradation trajectory is non-linear. Since the response function we chose does not depend on environmental forces, we do not need to define the continuous environmental force function.

Let us suppose further that the users as a whole adjust the level of usage of the infrastructure depending on its performance. For that, we introduce the demand function

$$d_f = \left(\frac{V - v_{min}}{v_{max} - v_{min}}\right)^4 \alpha \tag{31}$$

where $\alpha = 2.8 \times 10^7$ vehicles per year is the demand at the maximum performance. The chosen demand function links the infrastructure performance to the agent's balance. When performance is low, demand is low and the agent's revenue rate decreases which halts the growth of the agent's balance. Additional growth to the agent's balance is gained when the performance is restored by maintenance interventions.

Experiment 4. In this experiment, the principal reduces the inspection intervals from 4 years to 2 years. Then, the agent perform more frequent maintenance interventions. Under this circumstances, his utility shows a 38.8% increase and the real performance increased 15.2%. also, the principal nearly doubled the accuracy of $\hat{V}$ but its absolute value only increased slightly. Finally, the higher $\hat{V}$ was counteracted by the cost increase caused by the additional inspections, therefore u_P remained unchanged.

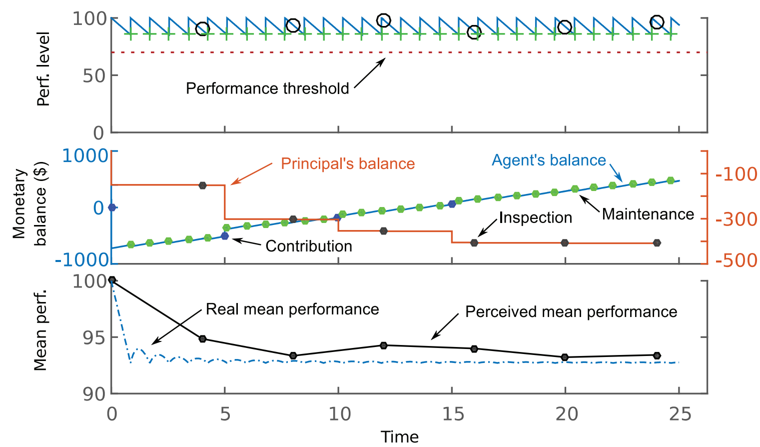

Fig. 9. Evolution of variables of interest (performance level, players' monetary balance and real and perceived mean performance) from the realization of experiment 1.

Experiment 5. In this case, the principal chooses an inspection strategy in which the time interval between inspections is exponentially distributed with parameter $\lambda = 0.5$. Then, the agent—who is still using the adaptive voluntary maintenance strategy which assumes the inspections occur at regular times—is unable to estimate correctly the next inspection and is therefore detected and punished several times.

As a result, the perceived mean performance is less biased than in the previous experiment. The real mean performance does not change significantly. The random inspection times produce variability in the outcome of the game. Fig. 10 shows a single realization from experiments 5 and 6; and Fig. 11 shows the dispersion plot of u_A and u_P for 500 realizations of experiments 5 and 6.

Experiment 6. This experiment introduces natural hazard with a strategy that generates shocks at exponentially distributed time intervals with $\lambda = 0.5$ and an environmental force that is distributed log-normal with mean 10 and $COV = 0.5$. Suppose the shock environmental force is given as an imposed displacement in cm/m: centimeters of vertical displacement in one meter of longitudinal distance along the road. Such displacements can be the result of seismic activity or the result of an unstable subgrade. The discrete response is defined by the function

$$r_d(\hat{f}, V) = \begin{cases} 0 & \frac{V}{\hat{f}} \geq \frac{10}{5} \\ V - \frac{10}{5}\hat{f} & \text{otherwise} \end{cases} \tag{32}$$

As expected, random shocks and inspection times produce variability in the outcome of the game (see Fig. 11). Therefore, the agent is worse off with $\mathbb{E}[u_A]$ reduced by 103.9% and $Var[u_A]$ increased notably; However, $\mathbb{E}[u_P]$ only drops 15.2%.

7.4. Analysis of experiments

The simulation model was used to evaluate the outcome of the game under specific strategy profiles and problem parameters which were described in each experiment. For deterministic strategy profiles—as in experiments 1 through 4—a single realization is needed to observe the dynamic behavior of the system (see Fig. 10). In the case of strategies that include a stochastic component—as in experiments 5 and 6—many realizations have to be executed to compute the distribution of utilities (see Fig. 11) and other indicators. In addition of the quantitative results, the experiments also show interesting features of the model:

- *Strategy adaptability:* Adaptive strategies may be able to exploit other strategies, particularly those that are non-adaptive. For instance, the voluntary maintenance strategy of experiment 2 was able to take advantage of the regularity of the inspection strategy to synchronize with it and delay maintenance works as long as possible.
- *Imperfect assessment:* The knowledge of output is only approximate, and thus it corresponds to a principal's belief. For instance, in experiment 2 the agent was able to increase his payoff at the principal's expense, even though she didn't perceive the full extent of her loss (observe the gap between the estimated and the real mean performance in the transition from experiment 1 to experiment 2).
- *Incentive shift:* The goodness of a strategy may depend on problem parameters. For example, in experiment 4 the agent increased his utility even though he had to do more frequent maintenance interventions. When experiment 3 introduced a non-linear deterioration and a revenue function dependent on performance, the agent's optimal strategy shifted. There appeared an incentive for the agent to keep the performance level high because it translated into more revenue to his monetary balance.
- *Strategies success is contingent:* The goodness of a player's strategy may also depend on the strategy chosen by the other players. For example, in experiment 4, when the principal increased his inspection frequency, the agent was able to adapt to it using the same strategy. The real mean performance increased which causes the estimation of the principal's utility to be more accurate. Here we observe how the reliability of the principal's information depends on both the inspection strategy and maintenance strategy. The change in the inspection strategy in experiment 5 made the principal's estimation of the mean performance more accurate without changing too much the real mean.
- *Effects of natural hazard:* Uncertainty and shock-based degradation affect the players' utilities. Experiment 6 shows how a hazardous environment greatly diminishes the utility of the agent (who is bearing all the risk due to natural hazards) while lowering both the principal's utility and the real mean performance to a lesser degree.

8. Discussion

This section will address two important aspects of the approach we propose: a discussion about its validation and ways to improve our model.

8.1. Validation

Validation is clearly one of the main issues in engineering modeling. As the complexity of problem increases, the issue of

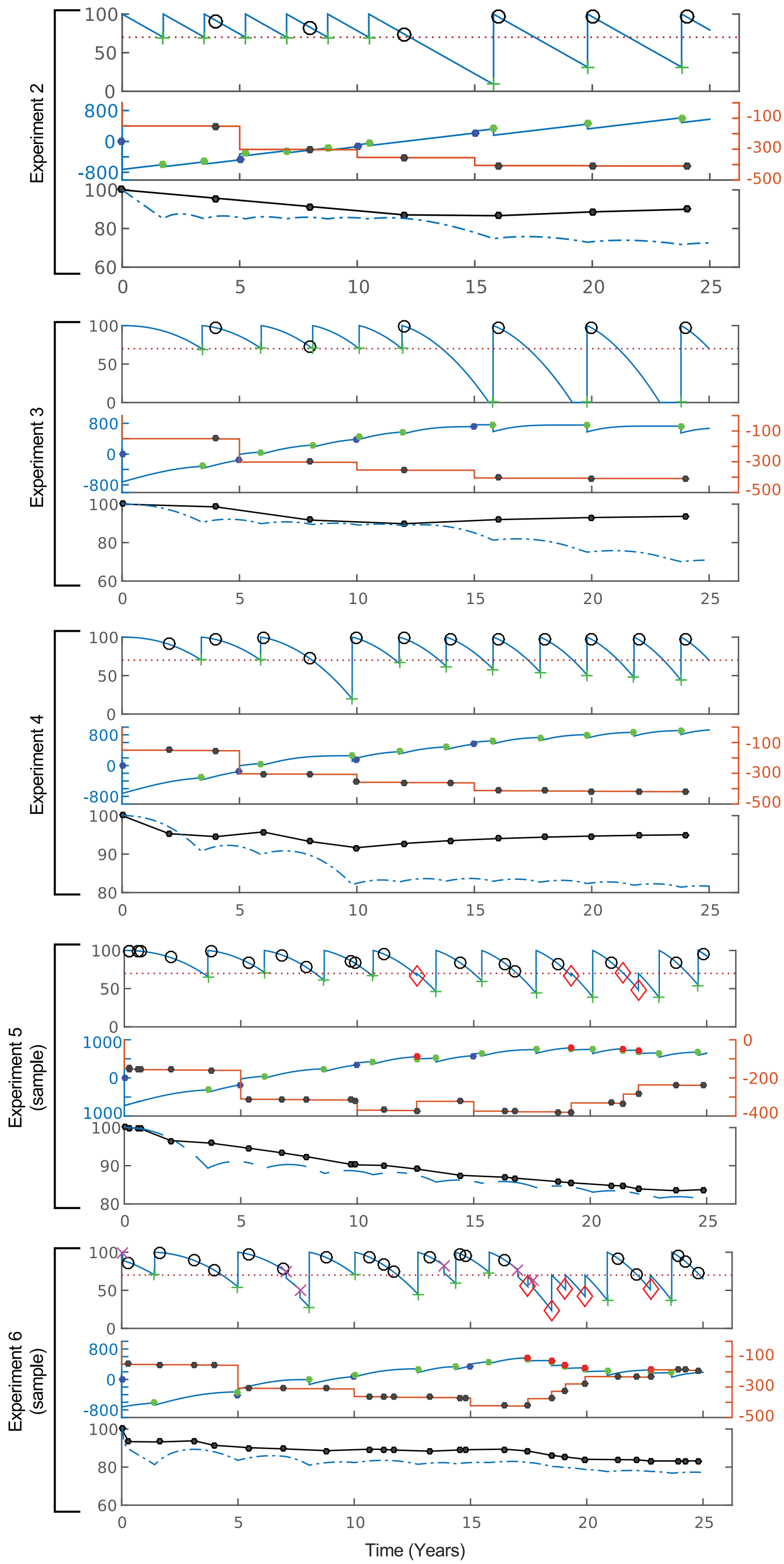

Fig. 10. Dynamic output from experiments 2 to 6. Plots from experiments 5 and 6 are showing a typical realization of the stochastic process.

validation becomes a central element of any model. Richiardi, Leombruni, Saam, and Sonnessa (2006) mention five aspects of validation that must be checked: (1) theory: validity of theory relative to the real-world system, (2) model: validity of the model relative to the theory, (3) program: validity of the simulation program relative to the model, (4) operational: validity of the concepts within the model relative to measurable indicators of the real-world system, (5) empirical: the validity of model indicators relative to empirical observations of the real-world system.

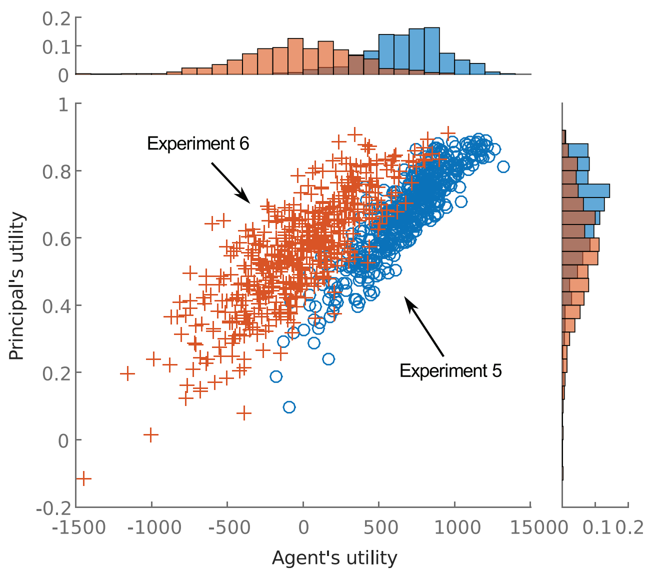

Fig. 11. Results of 500 realizations of Experiment 6. The dispersion patterns are synthesized by normalized histograms.

Keeping in mind this taxonomy, let's first say all theoretical approaches that have been discussed and that constitute the core of the model are well known and many instances of their merits can be easily found in the references provided; this complies with the first aspect. Furthermore, throughout the paper we progressively showed how our model is built upon the theoretical bases mentioned above; which validates the fact that the model is built on solid theoretical basis. In our model we use also a solid theoretical background to achieve cohesive representation of the real-world system, which validates the model with respect to the theory. Regarding the validity of the simulation program relative to the model, the agent-based model has been tested by running various problem instances that illustrate and show coherence. Finally, with respect to the validity of model indicators we are currently working on modeling actual cases found in the real world.

In summary, our model was designed to demonstrate that there is a relationship between the economic game, the contractual design, the physical infrastructure system and the natural environment, all of which jointly determine the output (success or failure) of such delegation. The experiments conducted in Section 7 show these relationships, thus validating the main hypotheses that motivated this study.

8.2. Future work

We believe that the work proposed in this paper has a great potential for improvement; some areas of future work include:

Simultaneous adverse selection and moral hazard. The difficulty of extracting private information from the firms to choose a good maintenance cost function ψ hints at the possibility of analyzing the problem as a principal-agent game with simultaneous adverse selection and moral hazard (Laffont & Martimort, 2009, Ch. 7.1); at the cost, however, of further complicating the model definition.

Data and validation of the descriptive model. With access to quality data, the conjecture that this model is descriptive can be tested. Comments on how to perform this test were given in the previous subsection.

Exploration of strategy space and parameter space. It is important testing the strategy space, searching for principal's strategies that are robust when faced with different kinds of agent's strategies. It is also important to explore how the outcome of strategy profiles become efficient or not as the problem parameters vary.

Risk-sharing scheme. Although the 'partnership' in a PPP is actually a contractual relationship, it has the peculiarity that the agent shares risks with the principal. Besides the elements that compose the contract in our model (contract duration, payment schedule, revenue function, performance threshold and penalty policy) a very important part of a PPP contract is the risk-sharing scheme. This scheme should unambiguously describe how costs and responsibilities will be shared among principal and agent for every possible state of the world. It is a very important part of a contract because it deals with events that are fundamentally uncertain, such as traffic demand, natural disasters, political instability, which may need a large amount of money to be resolved. For this reason, it has a strong influence in the final utilities of both players. This not captured in the present model (i.e., the agent fully pays for all maintenance works) due to several difficulties. One of them being the inability of an ignorant principal within the model to trace causal relationships based on incomplete and imperfect information. For instance, who should bear responsibility for a violation detected soon after a shock? How would the principal determine if it was mainly the result of the agent's negligence or the result of sudden shocks, given that she is ignorant about both agent's actions and shock events? A specific heuristic policy would be necessary to solve this issue.

Spatial awareness. Agents in our model and the infrastructure system itself exist in a temporal dimension, but they lack the concept of spatial dimensions altogether. The inclusion of an interaction topology, such as a network, would allow a more realistic representation of environmental pressures and the constraint that the logistics of movement imposes upon entities in the real world.

Optimization. The most ambitious application of the simulation model we developed is the possibility of finding what the principal should choose as (1) contract duration, (2) payment schedule, (3) revenue function, (4) performance threshold, (5) penalty strategy (i.e., penalty policy) and (6) inspection strategy, in order to achieve a stochastic maximization of u_P while ensuring that the probability of u_A being greater than a reservation utility $\bar{u}$ is at least some specified reliability value, for a wide range of agent's strategies and a given set of problem parameters. Even though the strategy space of the game is large, a set of predefined and parameterized strategies could be arranged to feed an optimization program capable of arriving to a strategy profile that solves the principal's problem within that particular subset of the strategy space. A program like this, would be an extension of the use case shown in Fig. 8, that implemented the bi-level optimization with their respective objectives and constraints. In Fig. 12 we present the use case of such program—with a government agency in the role of user/modeler.

9. Conclusions

Delegation plays a prominent role in the procurement of infrastructure systems. The delegation of tasks to a self-interested entity coupled with random changes in the environment creates a moral hazard problem. We showed that aggregate models have limitations in two main respects. First, they offer inappropriate account of how players' actions produce outcomes by proposing a

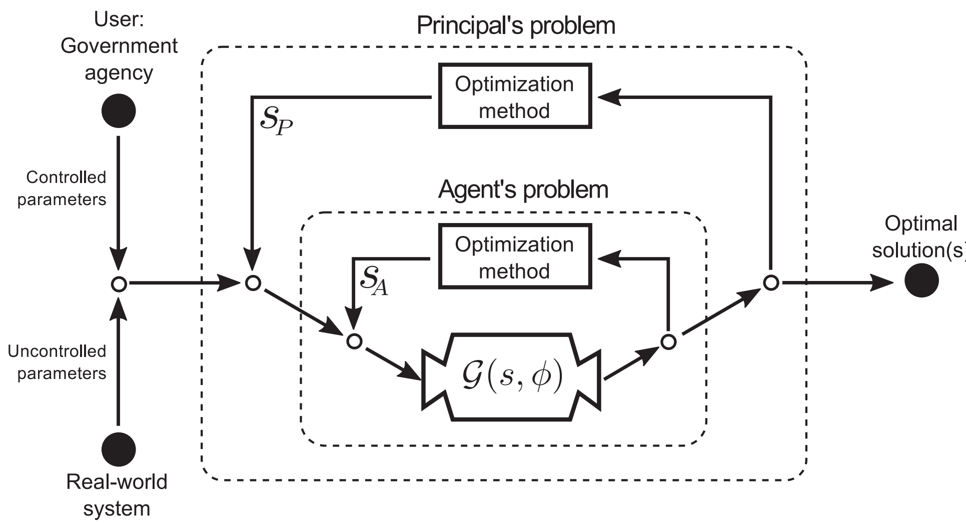

Fig. 12. Use case of optimization model.

functional relationship that is not suitable to express concrete operations and response of physical objects. Second, we pointed out that the assumption of the principal automatically observing the infrastructure's performance is not sensible for large infrastructure projects and thus we proposed a game that integrates inspections as the only way the principal can estimate her own utility, determine compliance of the performance threshold and deter shirking from the agent by the threat of penalties.

The main contributions of the proposed model can be summarized as follows:

- It is a novel approach to the problem of infrastructure development by modeling the interaction between different players regarding technical, economic and operational aspects. This recognizes that large infrastructure projects are the result of complex interactions and that their success depends on understanding them.
- It can evaluate more realistically the relationship between players' actions and their effect on the infrastructure system. This yields a better representation of the player's payoffs that result from their chosen strategies, while embracing the complexity in which the process is embedded. This is necessary to apply the concepts of equilibrium and optimization to arrive at desired values of static final quantities such as expected utilities.
- It can monitor the dynamic evolution of all the components within a game realization. This can be potentially used to create optimized contract designs that are able to control dynamic aspects such as the trajectory of performance value or any other component of the system.

Physical infrastructure is one of the pillars of productivity and prosperity of a country (Schwab & Sala-i Martin, 2014). We think the issues we address in this paper are very relevant for any organized society. We also think that the representation of a socio-technical system (e.g., the development of infrastructure under delegation) with a computational model is promising in the face of the increased capacity and speed of computers to deal with demanding simulations and optimization algorithms.

The present work is motivated by the premise that the success of the development of public works of infrastructure is collectively determined by the technological capabilities offered by engineering skills which come in the form of design, construction and maintenance, by the economic relationship and contractual agreements between the entities who commit to the task and by the natural environment in which the process takes place. At the scale of complex projects, all these factors are interrelated; they are decisive for efficiency and should not be treated in complete isolation. We hope our contribution may encourage more efforts to unify these perspectives so that future implemented policies in infrastructure development may bring about positive changes more effectively.

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Appendix A. Notation

Symbol	Description		
Section 2			
q	Output	θ	Noise introduced by Nature
w	Wage transferred to the agent	u_A	Agent's utility
e	Agent's effort	u_P	Principal's utility
Section 3			
V	Performance of infrastructure	r_f	Revenue rate function
t_m	Contract duration	k	Performance threshold
h	Payment schedule	s_L	Penalty fee strategy
Section 5			
b_A	Agent's monetary balance	Ξ	Shock action set
b_P	Principal's monetary balance	χ_i	Information known to ith player
Γ	Set of states of the game	d_f	Demand function
γ	Current state of the game	f_c	Continuous environmental force
A_1	Agent's action set	r_c	Continuous response function
A_2	Principal's action set	r_d	Discrete response function
A_3	Nature's action set	ψ	Maintenance cost function
O	Null action set	a_i	An action of the ith player
Λ	Voluntary maintenance action set	s_i	A strategy of the ith player
M	Mandatory maintenance action set	$\mathbf{a}$	An action profile
Υ	Performance space of infrastructure	$\mathbf{s}$	A strategy profile
Υ_ω	Maintenance space	S_i	Strategy set of the ith player
C	Contract offer action set	S	Strategy space of the game
$\mathcal{I}$	Inspection action set	ϕ	Problem parameters
L	Penalty fee action set	$\mathcal{G}$	Game realization function

Appendix B. Algorithms

Input:
- Parameters from table 3
- Strategy profile s

/* Initial state */
$V \leftarrow v_0$
$b_A \leftarrow b_A^0$
$b_P \leftarrow 0$
$\gamma \leftarrow 0$
$x \leftarrow (V, b_A, b_P, \gamma)$
$(t, j) \leftarrow (0, 0)$
/* Contract proposal */
$a_P \leftarrow s_2(\chi_2^{(t,j)}, \gamma)$
$(t_m, h, r_f, k) \leftarrow a_P$
$\gamma \leftarrow 1$
/* Beginning of interaction */
while $t \leq t_m$ **do**
  /* Payment schedule */
  $b_A \leftarrow b_a + h(t)$
  $b_P \leftarrow b_P - h(t)$
  $j \leftarrow j + 1$
  $x \leftarrow (V, b_A, b_P, \gamma)$
  /* Proactive actions */
  $a \leftarrow s(\chi^{(t,j)}, \gamma)$
  $(a_A, a_P, a_N) \leftarrow a$
  if $(a_A, a_P, a_N) = (\tilde{o}, \tilde{o}, \tilde{o})$ **then**
    /* The system flows */
    $b_A \leftarrow b_A + r_f(d_f(V, t)) * dt$
    $V \leftarrow V + r_c(f_c(t), d_f(V, t), V, t) * dt$
    $t \leftarrow t + dt$
    $x \leftarrow (V, b_A, b_P, \gamma)$
  else
    /* The system jumps */
    JumpMap ▷ *(See algorithm 2)*
  end
end

Algorithm 1: Game realization: evolution of hybrid system.

if $a_N \in \Xi$ **then**
  /* Shock */
  $f_s \leftarrow a_N$
  $V \leftarrow V - r_d(fs, V, t)$
  $j \leftarrow j + 1$
  $x \leftarrow (V, b_A, b_P, \gamma)$
else if $a_A \in \Lambda$ **then**
  /* Vol. maint. */
  $(v_i, v_f) \leftarrow a_A$
  $b_A \leftarrow b_A - \psi(a_A)$
  $V \leftarrow v_f$
  $j \leftarrow j + 1$
  $x \leftarrow (V, b_A, b_P, \gamma)$
else
  /* Inspection */
  $b_P \leftarrow b_P - c_\iota$
  $j \leftarrow j + 1$
  $x \leftarrow (V, b_A, b_P, \gamma)$
  if $V < k$ **then**
    $\gamma \leftarrow 2$
    /* Penalty fee */
    $a_P \leftarrow s_2(\chi_2^{(t,j)}, \gamma)$
    $p \leftarrow a_P$
    $b_A \leftarrow b_A - p$
    $b_P \leftarrow b_P + p$
    $j \leftarrow j + 1$
    $x \leftarrow (V, b_A, b_P, \gamma)$
    $\gamma \leftarrow 3$
    /* Mand. maint. */
    $a_A \leftarrow s_1(\chi_1^{(t,j)}, \gamma)$
    $(v_i, v_f) \leftarrow a_A$
    $b_A \leftarrow b_A - \psi(a_A)$
    $V \leftarrow v_f$
    $j \leftarrow j + 1$
    $x \leftarrow (V, b_A, b_P, \gamma)$
    $\gamma \leftarrow 1$
  end
end

Algorithm 2: JumpMap.

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